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COMPUTER TECHNIQUES FOR ELECTROMAGNETICS

EDITED BY
R. MITTRA

University of Illinois, Urbana, Illinois

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Preface

Computer Electromagnetics is a relatively new and rapidly growing field. Yet, to date most of the publications on the subject have appeared only in technical journals or reports, some of which are not readily accessible. It is hoped that this book will serve to fill the existing need for a comprehensive text on the subject of computer techniques for solving a wide class of practical problems in applied electromagnetics, e.g. calculation of current distribution on antennas; computation of radar scattering from conducting or dielectric bodies; evaluation of discontinuity effects in waveguides and arrays; numerical reconstruction of wavefronts and aperture fields; and so on.

The manuscript is an outgrowth of a text originally prepared for a seminar series on the same topic offered at the University of Illinois on October 1970. Each of the seminar speakers for the course was asked to provide written notes describing the content of his lecture. These were later edited and put together in the form of the present text. The short course was repeated in 1971, with only a slight change in the contents, at Copenhagen under the auspices of the Technical University of Denmark and the Danish Engineering Society. The notes were well received on both of these occasions and provided encouragement to the authors for completing the monumental task of revising, proofing and editing their contributions.

During the course of the preparation of the manuscript the authors have received encouragement and moral support from many individuals and organizations, too numerous to list here on individual basis. Nevertheless, the authors would like to take this opportunity to thank their colleagues for the generous help and suggestions they have received that have undoubtedly left indelible marks on the text and have served to improve it.

The editor (R. M.) would like gratefully to acknowledge collaboration he has received from his co-authors, even through the periods of disappointing delays that severely taxed their patience.

The manuscript was very ably reviewed for the publisher by Professor Alex Cullen of University College, London. His comments and criticisms on the script were most welcome and have helped to unify the presentation and to improve the contents of the book.

The final editing of the book was carried out during a sabbatical leave of absence from the University of Illinois when the editor (R. M.) was at the Technical University of Denmark, Lyngby, Denmark. He is particularly grateful to Professor H. L. Knudsen, Head of Laboratory for Electromagnetic Theory, for the facilities and support provided him during the six-month period of 1971–2. Needless to say, the bulk of the editing work and the writing of Chapters 6 and 7 of the book were carried out at the University of Illinois and it is a pleasure to acknowledge the support of the Electrical Engineering Department received during the years 1970–2.

University of Illinois
Urbana, Illinois

Raj Mittra